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## FERMI-DIRAC DISTRIBUTIONS FOR QUARK PARTONS

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### Abstract

We propose to use Fermi-Dirac distributions for quark and antiquark partons. It allows a fair description of the  $x$ -dependence of the very recent NMC data on the proton and neutron structure functions  $F_2^p(x)$  and  $F_2^n(x)$  at  $Q^2 = 4 \text{ GeV}^2$ , as well as the CCFR antiquark distribution  $x\bar{q}(x)$ . We show that one can also use a corresponding Bose-Einstein expression to describe consistently the gluon distribution. The Pauli exclusion principle, which has been identified to explain the flavor asymmetry of the light-quark sea of the proton, is advocated to guide us for making a simple construction of the polarized parton distributions. We predict the spin dependent structure functions  $g_1^p(x)$  and  $g_1^n(x)$  in good agreement with EMC and SLAC data. The quark distributions involve some parameters whose values support well the hypothesis that the violation of the quark parton model sum rules is a consequence of the Pauli principle.

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# 1 Introduction

Many years ago Feynman and Field made the conjecture<sup>[1]</sup> that the quark sea in the proton may not be flavor symmetric, more precisely  $\bar{d} > \bar{u}$ , as a consequence of Pauli principle which favors  $d\bar{d}$  pairs with respect to  $u\bar{u}$  pairs because of the presence of two valence  $u$  quarks and only one valence  $d$  quark in the proton. This idea was confirmed by the results of the NMC experiment<sup>[2]</sup> on the measurement of proton and neutron unpolarized structure functions,  $F_2(x)$ . It yields a fair evidence for a defect in the Gottfried sum rule<sup>[3]</sup> and one finds

$$I_G = \int_0^1 \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = 0.240 \pm 0.016 \quad (1)$$

instead of the value  $1/3$  predicted with a flavor symmetric sea, since we have in fact

$$I_G = \frac{1}{3}(u + \bar{u} - d - \bar{d}) = \frac{1}{3} + \frac{2}{3}(\bar{u} - \bar{d}). \quad (2)$$

A crucial role of Pauli principle may also be advocated to explain the well known dominance of  $u$  over  $d$  quarks at high  $x$ ,<sup>[4]</sup> which explains the rapid decrease of the ratio  $F_2^n(x)/F_2^p(x)$  in this region. Let us denote by  $q^\uparrow(q^\downarrow)$ ,  $u$  or  $d$  quarks with helicity parallel (antiparallel) to the proton helicity. The double helicity asymmetry measured in polarized muon (electron) - polarized proton deep inelastic scattering allows the determination of the quantity  $A_1^p(x)$  which increases towards one for high  $x$ ,<sup>[5]</sup> suggesting that in this region  $u^\uparrow$  dominates over  $u^\downarrow$ , *a fortiori* dominates over  $d^\uparrow$  and  $d^\downarrow$ , and we will see now, how it is possible to make these considerations more quantitative. Indeed at  $Q^2 = 0$  the first moments of the valence quarks are related to the values of the axial couplings

$$u_{\text{val}}^\uparrow = 1 + F, \quad u_{\text{val}}^\downarrow = 1 - F, \quad d_{\text{val}}^\uparrow = \frac{1 + F - D}{2}, \quad d_{\text{val}}^\downarrow = \frac{1 - F + D}{2}, \quad (3)$$

so by taking  $F = 1/2$  and  $D = 3/4$  (very near to the quoted values<sup>[6]</sup>  $0.477 \pm 0.011$  and  $0.755 \pm 0.011$ ) one has  $u_{\text{val}}^\uparrow = 3/2$  and  $u_{\text{val}}^\downarrow = 1/2$  which is at the center of the rather narrow range  $(d_{\text{val}}^\uparrow, d_{\text{val}}^\downarrow) = (3/8, 5/8)$ . The abundance of each of these four valence quark species, denoted by  $p_{\text{val}}$ , is given by eq. (3)

and we assume that the distributions at high  $Q^2$  “keep a memory” of the properties of the valence quarks, which is reasonable since for  $x > .2$  the sea is very small. So we may write for the parton distributions

$$p(x) = F(x, p_{\text{val}}) \quad (4)$$

where  $F$  is an increasing function of  $p_{\text{val}}$ . The fact that the dominant distribution at high  $x$  is just the one corresponding to the highest value of  $p_{\text{val}}$ , gives the correlation *abundance* - *shape* suggested by Pauli principle, so we expect broader shapes for more abundant partons. If  $F(x, p_{\text{val}})$  is a smooth function of  $p_{\text{val}}$ , its value at the center of a narrow range is given, to a good approximation, by half the sum of the values at the extrema, which then implies<sup>[7]</sup>

$$u^\downarrow(x) = \frac{1}{2}d(x). \quad (5)$$

This leads to

$$\Delta u(x) \equiv u^\uparrow(x) - u^\downarrow(x) = u(x) - d(x) \quad (6)$$

which allows to relate the contribution of  $u$  quark to  $g_1^p(x)$ , the proton polarized structure function, to the contributions of  $u$  and  $d$  quarks to  $F_2^p(x) - F_2^n(x)$  i.e.

$$xg_1^p(x)\Big|_u = \frac{2}{3}(F_2^p(x) - F_2^n(x))\Big|_{u+d}. \quad (7)$$

Moreover, since  $\Delta d_v = (\frac{F-D}{2F})\Delta u_v = -1/4\Delta u_v$  and  $e_d^2 = 1/4e_u^2$ , we expect a small negative correction to  $g_1^p(x)$  from  $d$  quarks and therefore

$$xg_1^p(x) \simeq \frac{2}{3}(F_2^p(x) - F_2^n(x)), \quad (8)$$

at least in the  $x$  region dominated by valence quarks. This is confirmed by experiment<sup>[2,5]</sup>, which shows good agreement with eq. (8) for  $x \geq .2$ , with a slight overestimate of  $g_1^p(x)$  at small  $x$ , partly due to the fact that we have neglected the  $d$  quark contribution  $\Delta d$ , which is expected to be negative<sup>[7]</sup>. All these features are encouraging to take rather seriously the importance of Pauli principle and have led us to propose a new description of the quark

parton distributions in terms of Fermi-Dirac expressions as functions of the scaling variable  $x$  according to<sup>[8]</sup>

$$p(x) = \frac{f(x)}{e^{\frac{x-\tilde{x}(p)}{\bar{x}}} + 1} \quad . \quad (9)$$

Here  $\tilde{x}(p)$  plays the role of the “thermodynamical potential” for the fermionic parton  $p$  of a given flavor and helicity,  $\bar{x}$  is the “temperature” and  $f(x)$  is a weight function whose support is the open range  $(0, 1)$ . As we will see, we will be able to reproduce the NMC data<sup>[9]</sup> for  $F_2^p(x)$  and  $F_2^n(x)$  taken at  $Q^2 = 4 \text{ GeV}^2$ , and the antiquark data from neutrino deep inelastic scattering<sup>[10]</sup>. This will allow the determination of the small number of parameters entering in eq. (9). Keeping the same value for  $\bar{x}$ , we will also produce a gluon distribution  $G(x)$  in terms of a Bose-Einstein expression similar to eq. (9) and consistent with what is known from experiment<sup>[11,12]</sup>. We will find for  $\tilde{x}(u^\uparrow)$  a much larger value than for  $\tilde{x}(u^\downarrow)$  (i.e.  $\tilde{x}(u^\uparrow) \sim 4\tilde{x}(u^\downarrow)$ ) which suggests that the defect in the Gottfried sum rule is a consequence of the fact that the “ $u^\uparrow$  bus” is almost full and therefore the sea quarks have to fill up all available space in the “ $u^\downarrow$  bus” and in the “ $d$  bus”. This allows us to propose an interesting solution of the spin crisis, where we will have a fair description of  $g_1^p(x)$  obtained by EMC<sup>[5]</sup> and also of the neutron polarized structure function  $g_1^n(x)$  obtained very recently at SLAC<sup>[13]</sup>.

In the next section we present our new approach for parton distributions and we will specify our parameterization for  $u$  and  $d$  quarks, for antiquarks and for gluons. In section 3, by using the most recent and accurate available low  $Q^2$  unpolarized deep inelastic data, we will see that it is possible to complete the determination of all these distributions in terms of eight free parameters and we will also present our predictions for polarized deep inelastic scattering. In section 4, we will discuss shortly the outcoming results of this approach for some quark parton sum rules. We will give our concluding remarks in section 5.

## 2 New Approach to Parton Distributions

Let us first consider  $u$  quarks and antiquarks and we assume that  $u^\uparrow(x)$ ,  $u^\downarrow(x)$ ,  $\bar{u}^\uparrow(x)$  and  $\bar{u}^\downarrow(x)$  are expressed in terms of Fermi-Dirac distributions of the form given by eq. (9). The thermodynamical potential  $\tilde{x}(p)$  is a constant

parameter which is expected to be different for each parton  $p$ , whereas the temperature  $\bar{x}$  can be a universal constant and we will take for  $f(x)$  a simple form\*

$$f(x) = Ax^\alpha. \quad (10)$$

Concerning the  $d$  quarks and antiquarks, following our above arguments we will assume that

$$d(x) = \frac{u^\downarrow(x)}{1 - F} \quad (5')$$

which is a slight modification of eq. (5), to account for the fact that  $F$  is not exactly  $1/2$ . As also indicated above,  $\Delta d(x)$  is not very large and moreover the potentials associated to  $d^\uparrow(x)$  and  $d^\downarrow(x)$  are expected to satisfy the following constraints

$$0 < \tilde{x}(d^\downarrow) - \tilde{x}(u^\downarrow) \simeq \tilde{x}(u^\downarrow) - \tilde{x}(d^\uparrow) < \bar{x}. \quad (11)$$

Therefore this observation justifies, to a reasonable approximation, the following choice

$$\Delta d(x) = -kf(x) \frac{e^{\frac{x - \tilde{x}(u^\downarrow)}{\bar{x}}}}{\left(e^{\frac{x - \tilde{x}(u^\downarrow)}{\bar{x}}} + 1\right)^2}, \quad (12)$$

an expression with no new parameter, except the normalization factor  $k$  which will be fixed by requiring for the first moment

$$\Delta d = \Delta d_{\text{val}} = F - D. \quad (13)$$

We therefore assume that the  $d$  sea quarks are not polarized<sup>†</sup> and consistently we also take for the  $d$  antiquarks,

$$\bar{d}^\uparrow(x) = \bar{d}^\downarrow(x) \equiv \bar{u}^\downarrow(x). \quad (14)$$

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\*With this choice  $p(x)$  does not go to zero when  $x \rightarrow 1$ , but it has a fast decrease coming from the exponential in the denominator.

<sup>†</sup>Clearly whereas one needs a large negative polarization for  $u$  sea quarks and antiquarks, it is less crucial for the case of the  $d$ 's which were assumed to be slightly polarized in refs.[14] and [15].

Concerning the strange quarks we first take in accordance with the data<sup>[10]</sup>

$$s(x) = \bar{s}(x) = \frac{\bar{u}(x) + \bar{d}(x)}{4}. \quad (15)$$

Finally for the gluon distribution, for the sake of consistency, one should assume a Bose-Einstein expression given as

$$G(x) = \frac{16}{3} \frac{f(x)}{e^{\frac{x-\tilde{x}(G)}{\bar{x}}} - 1} \quad (16)$$

with the same temperature  $\bar{x}$ , a specific potential  $\tilde{x}(G)$  and where the factor  $16/3$  is just twice the ratio of the color degeneracies of gluon and quarks since  $G(x)$  is the unpolarized gluon distribution.

This completes the parameterization of all parton distributions we will use and we will now proceed to the determination of the eight parameters we have introduced by means of the description of low  $Q^2$  unpolarized deep inelastic scattering data. We will also give the predictions it leads to for the polarized structure functions  $g_1^p(x)$  and  $g_1^n(x)$ .

### 3 Unpolarized and Polarized Deep Inelastic Scattering

To determine our parameters we have used the most recent NMC data<sup>[9]†</sup> on  $F_2^p(x)$  and  $F_2^n(x)$  at  $Q^2 = 4 \text{ GeV}^2$ , together with the most accurate CCFR data on the antiquark distribution [10].

We get the following set of parameters

$$\bar{x} = 0.132, \quad A = 0.579, \quad \alpha = -0.845, \quad (17)$$

$$\tilde{x}(u^\uparrow) = 0.524, \quad \tilde{x}(u^\downarrow) = 0.143, \quad \tilde{x}(\bar{u}^\uparrow) = -0.216, \quad \tilde{x}(\bar{u}^\downarrow) = -0.141.$$

For the fraction of the total momentum carried by quarks and antiquarks we find

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<sup>†</sup>In ref.[9] the result quoted in eq.(1) has been slightly changed to  $I_G = 0.258 \pm 0.017$ .

$$\begin{aligned}\int_0^1 xu(x)dx &= 0.278, \quad \int_0^1 xd(x)dx = 0.075, \\ \int_0^1 x[\bar{u}(x) + \bar{d}(x) + s(x) + \bar{s}(x)]dx &= 0.084.\end{aligned}\tag{18}$$

We show the results of the fit for  $F_2^p(x) - F_2^n(x)$  and  $F_2^n(x)/F_2^p(x)$  in Figs. 1 and 2 and in Fig. 3 our prediction for the distribution of the antiquarks compared to the data<sup>[10]</sup>. The behavior of the antiquark distribution is correctly reproduced for  $x > 0.1$  but for small  $x$  values we don't have the fast increase shown by the data. This is due to our simplifying assumptions that the  $\tilde{x}$ 's and  $\bar{x}$  are taken to be constant. Actually when  $x \rightarrow 0$ , from Pomeron universality, one expects  $x\bar{u}(x) = x\bar{d}(x) \neq 0$ , contrary to the present situation where we took for quarks and antiquarks a universal  $f(x)$  such that  $xf(x)$  vanishes when  $x$  goes to zero. Clearly this is not adequate for antiquarks and could be improved in a more sophisticated version of this approach for example by allowing the  $\tilde{x}$ 's to depend on  $x$ . In particular, we underestimate  $\bar{u}$  and  $\bar{d}$  so we don't satisfy very accurately the obvious constraints

$$u - \bar{u} = 2 \quad \text{and} \quad d - \bar{d} = 1.\tag{19}$$

We now turn to the gluon distribution (see eq. (16) whose only free parameter  $\tilde{x}(G)$  has been fixed by the requirement that gluons carry the fraction of the total momentum not carried by quarks and antiquarks, that is 0.563, according to eq. (18). We find a very small value  $\tilde{x}(G) = -0.012$  and we display in Fig. 4 our prediction which is fairly consistent with some preliminary indirect experimental determination from direct photon production<sup>[11]</sup> and from neutrino deep inelastic scattering<sup>[12]</sup> at  $Q^2 = 5 \text{ GeV}^2$ .

Going back to quark distributions we can now test our approach by looking at the predictions we obtain for the polarized proton and neutron structure functions. So far, all the parameters have been fixed, except the normalization factor  $k$  of  $\Delta d$  (see eq. (12)) which was found to be  $k = 0.787$  after imposing eq. (13). The results at  $Q^2 = 4 \text{ GeV}^2$  are shown in Fig. 5 for  $g_1^p(x)$  and in Fig. 6 for  $g_1^n(x)$ . For  $g_1^p(x)$  the agreement with the data which corresponds to  $\langle Q^2 \rangle = 10 \text{ GeV}^2$ , is very satisfactory but no significant  $Q^2$  dependence has been observed by EMC<sup>[5]</sup>. For  $g_1^n(x)$  we have the general trend of the data at  $\langle Q^2 \rangle = 2 \text{ GeV}^2$ , which is small, because there is a strong cancellation between  $\Delta u(x)$  weighted by  $1/9$  and  $\Delta d(x)$  weighted by  $4/9$ .

However we fail to reproduce accurately the structure around  $x = 0.1$ , in spite of the fact that the driving contribution  $\Delta d(x)$  has a minimum at the correct value because  $\tilde{x}(u^\downarrow) = 0.143$  (see eq. (12)). These predictions are very encouraging and should certainly be improved.

## 4 Quark Parton Model Sum Rules

There is an intriguing consequence of the hypothesis that the defect in the Gottfried sum rule follows *only* from Pauli principle. In this framework since in the “ $u^\uparrow$  bus”, there is less available space than for the other partons, we expect that the defect in the Gottfried sum rule arises from the fact that the sea must contribute less to the  $u^\uparrow$  distribution. This implies from eq. (14)

$$\Delta u = \Delta u_{\text{val}} + \bar{u} - \bar{d} = 2F + \bar{u} - \bar{d} = 0.814 \pm 0.046, \quad (20)$$

which is smaller than  $\Delta u_{\text{val}}$  because  $\bar{d} > \bar{u}$ . Let us now consider the Bjorken sum rule<sup>[16]</sup> which reads

$$\begin{aligned} I_p - I_n &= \int_0^1 [g_1^p(x) - g_1^n(x)] dx = \frac{1}{6}(\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d}) \\ &= \frac{1}{6}(F + D) = \frac{1}{6}(\Delta u_{\text{val}} - \Delta d_{\text{val}}) \end{aligned} \quad (21)$$

where we have used eq. (3) for the last equality. Since in our approach  $\Delta d + \Delta \bar{d} = \Delta d_{\text{val}} = F - D$ , in order to satisfy the Bjorken sum rule we should have  $\Delta u + \Delta \bar{u} = \Delta u_{\text{val}} = 2F$  and therefore  $\Delta \bar{u}$  should be positive at variance with  $\Delta u_{\text{sea}}$ . Of course, given the fact that  $\Delta s = \Delta \bar{s} = 0$ ,<sup>[17]</sup> this leads for the parton Ellis-Jaffe sum rule<sup>[18]</sup>

$$\begin{aligned} I_p &= \int_0^1 g_1^p(x) dx = \frac{2}{9}(\Delta u + \Delta \bar{u}) + \frac{1}{18}(\Delta d + \Delta \bar{d}) \\ &= \frac{F}{2} - \frac{D}{18} = 0.196 \pm 0.006 \end{aligned} \quad (22)$$

to be compared with the EMC result  $0.126 \pm 0.015 \pm 0.009$ . However if one chooses



$$\Delta u = 2F + \bar{u} - \bar{d}, \quad \Delta \bar{u} = \Delta u_{\text{sea}} = \bar{u} - \bar{d}, \quad \Delta d = F - D, \quad \Delta \bar{d} = 0, \quad (23)$$

one gets

$$I_p = \frac{F}{2} - \frac{D}{18} + \frac{4}{9}(\bar{u} - \bar{d}) = 0.134 \pm 0.015$$

and

$$I_n = \frac{1}{3}F - \frac{2}{9}D + \frac{1}{9}(\bar{u} - \bar{d}) = -0.024 \pm 0.009 \quad (24)$$

in fair agreement with the proton EMC result<sup>[5]</sup> and with the neutron SLAC result<sup>[13]</sup> which is  $I_n = -0.022 \pm 0.011$ .<sup>§</sup>

This choice (eq. (23)) which is obtained by assuming the crucial role of Pauli principle for parton distributions implies that the theoretical framework for the derivation of the Bjorken sum rule should be reconsidered.

## 5 Concluding remarks

We have obtained a good description of the main features of unpolarized and polarized deep inelastic scattering by using Fermi-Dirac distributions for quark and antiquark partons. This supports the idea, previously proposed<sup>[7]</sup>, that Pauli exclusion principle plays a crucial role in deep inelastic phenomena and leads to the intriguing conclusion that partons interact incoherently but are bound to obey Fermi-Dirac statistics in the  $x$  variable. Concerning the parameters of these distributions, we have found a universal temperature  $\bar{x}$  for quarks, antiquarks and gluons, whereas the thermodynamical potentials  $\tilde{x}(p)$  are positive for quarks, negative for antiquarks and small for gluons. As expected, we find that  $\tilde{x}(u^\uparrow)$  dominates over all the other ones, so the " $u^\uparrow$  bus" is rather full which is naturally related to the defect of some quark parton model sum rules. In its present form, our approach fails to reproduce the fast increase of the antiquark distribution at very low  $x$  and this deserves some further comments. One obvious reason is that within

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<sup>§</sup>A much less accurate CERN experiment<sup>[19]</sup> leads to a different result, i.e.  $I_n = -0.08 \pm 0.04 \pm 0.04$ .

the parton model approximation the distributions are depending on  $x$  only, not on  $Q^2$ , and  $\bar{x}$  and  $\tilde{x}(p)$  were taken to be constants. Instead, we could have assumed that they depend also on the final hadronic invariant mass  $(P + q)^2 = M^2 + Q^2((1 - x)/x)$  which varies rapidly at small  $x$ . We may observe that this fast increase of the distributions at small  $x$  was advocated in the framework of the multiperipheral approach of deep inelastic scattering, where a singular behaviour  $x^{-3/2}$  was suggested<sup>[20]</sup> and this trend was recently observed in the data<sup>[21]</sup> for the first time. In this case one can speak pictorially of a liquid of partons for which the Fermi-Dirac gas description is inadequate. The parton distributions contain two phases, a gas contributing to the non singlet part which dominates at moderate and large  $x$  and a liquid contributing to the singlet part which prevails at low  $x$ . This is not a new situation and it corresponds to the well known two components picture, with ordinary Regge trajectories related to resonances and the Pomeron. Finally, we would like to add that we have been recently aware of a work<sup>[22]</sup> where deep inelastic structure functions are calculated in a statistical model of the nucleon considered as a gas of quarks and gluons in the framework of the MIT bag model. Despite the common reference to quantum statistics, the two approaches are rather different and in ref.[22] only unpolarized distributions have been considered.

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## Figure Captions

Fig.1 The difference  $F_2^p(x) - F_2^n(x)$  at  $Q^2 = 4 \text{ GeV}^2$  versus  $x$ . Data are from ref.[9] and the solid line is the result of our fit.

Fig.2 The ratio  $F_2^n(x)/F_2^p(x)$  at  $Q^2 = 4 \text{ GeV}^2$  versus  $x$ . Data are from ref.[9] and the solid line is the result of our fit.

Fig.3 The antiquark contribution  $x\bar{q}(x) = x\bar{u}(x) + x\bar{d}(x) + x\bar{s}(x)$  at  $Q^2 = 3 \text{ GeV}^2$  (open circles) and  $Q^2 = 5 \text{ GeV}^2$  (full triangles) versus  $x$ . Data are from ref.[10] and solid line is the result of our fit.

Fig.4 The gluon distribution  $xG(x)$  at  $Q^2 = 5 \text{ GeV}^2$  versus  $x$ . Data are from ref.[11] (area between dashed lines) and ref.[12] (area between small dashed lines) and solid line is the result of our calculation at  $Q^2 = 4 \text{ GeV}^2$ .

Fig.5  $xg_1^p(x)$  at  $\langle Q^2 \rangle = 10 \text{ GeV}^2$  versus  $x$ . Data are from ref.[5] and solid line is our prediction at  $Q^2 = 4 \text{ GeV}^2$ .

Fig.6  $xg_1^n(x)$  at  $\langle Q^2 \rangle = 2 \text{ GeV}^2$  versus  $x$ . Data are from ref.[13] and solid line is our prediction at  $Q^2 = 4 \text{ GeV}^2$ .

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